Question 1 (99%). Choose the most correct answer:

$n\pi$ $\quad n^2 \quad \pi$

(1) The series $\sum_{n=1}^{\infty} n \tan^{-1}(\frac{2}{n})$

(a) converges to 0
(b) converges to 2
(c) converges absolutely
(d) diverges

(2) The series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1.3.5\cdots(2n-1)}{(2n-1)!}$ $= \sum_{n=0}^{\infty} (-1)^n \frac{2n}{2n!}$

(a) converges conditionally
(b) converges absolutely
(c) diverges
(d) diverges absolutely

$= \sum_{n=0} (-1)^n \cdot \frac{}{(2}$

Positi

$\to 0$

decr

Conver

(3) The sum of the Maclaurin series $\pi - \frac{\pi^3}{3!} + \frac{\pi^5}{5!} - \cdots + \frac{(-1)^n \pi^{2n+1}}{(2n+1)!} + \cdots$ is

$\sin(\pi) = 0$

(a) 1

(b) 0

(c) -1

(d) e

(4) The interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(2x+3)^n}{\sqrt{n}}$ is

$\lim_{n\to\infty} \left| \frac{(2x+3)^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{(2x+3)^n} \right|$

(a) $-2 < x < -1$

(b) $-2 \le x \le -1$

(c) $-2 < x \le -1$

(d) $-2 \le x < -1$

$\sum_{n=1}^{\infty} \frac{(-2+3)^n}{\sqrt{n}} = \sum \frac{(1)^n}{\sqrt{n}}$ $\boxed{-1}$ div

$\boxed{-2}$ $\sum \frac{(-1)^n}{\sqrt{n}}$ A.S.T $\boxed{-2}$ conv

$= \lim_{n\to\infty} |(2x+3)|$

$|2x+3| \cdot 1 < 1$

$-1 < 2x+3 < 1$

$-4 < 2x < -2$

$-2 < x < -1$

(5) The interval of convergence of the series $\sum_{n=1}^{\infty} \left(\frac{\sqrt{x-1}-1}{2}\right)^n$ is

$\lim_{n\to\infty} \left|\frac{\sqrt{x-1}-1}{2}\right|$

(a) $1 \le x < 10$

(b) $1 < x < 10$ $\quad \left|\frac{\sqrt{x-1}-1}{2}\right| < 1$

(c) $2 \le x < 10$

(d) $2 < x < 10$

$-2 < \sqrt{x-1} - 1 < 2$

$-1 < \sqrt{x-1} < 3$

$1 < x-1 < 9$

$2 < x < 10$ $\boxed{2}$ $\sum \left(\frac{0}{2}\right)^n$ converge $\boxed{10}$

(6) $\lim_{n\to\infty} \frac{\ln(n^2)}{n}$

(a) ∞

(b) $-\infty$ $\quad = \lim \frac{(n^2)'}{n^2}$

(c) 0

(d) DNE $\quad \frac{2n}{n^2} = \frac{2}{n}$

(7) The sequence $a_n = (-1)^n \cos(n\pi)$ $= (-1)^n (-1)^n$

(a) converges to 1 $\quad = (1)^n = 1$

(b) converges to -1

(c) converges to 0 $\quad a_n = 1$

(d) diverges

$1, 1, 1, \ldots \to \boxed{1}$

(8) The sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^n 2^{n+1}}{5^n}$ is

$\sum_{n=1}^{\infty} \frac{(-1)^n \; 2^n \times 2}{5^n}$

(a) $\frac{5}{7}$

(b) $\frac{-4}{7}$ $\quad = 2 \times \sum_{n=1}^{\infty} \left(\frac{-2}{5}\right)^n$ $\quad |r| < 1$

(c) $\frac{5}{3}$

(d) $\frac{-4}{3}$

$= 2 \times \left(\frac{-2/5}{1 + \frac{2}{5}}\right) = 2 \times \frac{-2/5}{7/5} = \frac{-4}{7}$

$e^{(1+\frac{2}{n})^n}$

$\lim_{n\to\infty}$

(9) The sequence $\{a_n = e^{(1+\frac{2}{n})^n}\}$

(a) converges to e^e

(b) converges to 1

(c) converges to e^{e^2}

(d) converges to e^{2e}

$\ln(a_n) = (1+\frac{2}{n})^n$

$\lim \ldots = \lim \text{''} = e^2$

$e^{\ln a_n} = \boxed{e^{e^2}}$

(10) If $\sum_{n=0}^{\infty} r^{-n} = \frac{5}{4}$, then $r =$

(a) 9

(b) $\frac{9}{5}$

(c) 5

(d) $\frac{1}{9}$

$\sum_{n=0}^{\infty} \left(\frac{1}{r}\right)^n = \frac{1}{1-\frac{1}{r}} = \frac{5}{4}$

$\frac{4}{5} = 1 - \frac{1}{r} \Rightarrow \frac{1}{r} = 1 - \frac{4}{5}$

$\frac{1}{r} = \frac{1}{5} \Rightarrow \boxed{r=5}$

(11) If we estimate e^{-x} by $1 - x + \frac{x^2}{2}$, for $0 < x < 0.4$, then

(a) $e^{-x} \geq 1 - x + \frac{x^2}{2}$

(b) $e^{-x} = 1 - x + \frac{x^2}{2}$

(c) $e^{-x} \leq 1 - x + \frac{x^2}{2}$

(d) None of the above

$e^{-x} = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$

$E < 0$

$L - P < 0$

$L < P$

(12) The Taylor series generated by $f(x) = e^x$ at $a = 9$ is

(a) $\sum_{n=0}^{\infty} \frac{e^9(x-9)^n}{(n+1)!}$

(b) $\sum_{n=0}^{\infty} \frac{e^9(x-9)^{(n+1)}}{(n+1)!}$

(c) $\sum_{n=0}^{\infty} \frac{e^9(x-9)^{(n+1)}}{n!}$

(d) $\sum_{n=0}^{\infty} \frac{e^9(x-9)^n}{n!}$

$\sum_{n=0}^{\infty} \frac{(x-a)^n}{n!} = \sum_{n=0}^{\infty} \frac{(x-9)^n}{n!}$

e^9

(13) If $a_n > 0$ and $b_n > 0$ for all $n > N$ and $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$, then

(a) If $\sum a_n$ diverges, then $\sum b_n$ diverges.

(b) If $\sum b_n$ diverges, then $\sum a_n$ diverges.

(c) If $\sum b_n$ converges, then $\sum a_n$ converges.

(d) The series $\sum a_n$ and $\sum b_n$ both converge or both diverge.

(14) The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$

der $\left(\lim \frac{1}{\ln(n+1)}\right.$ Positive decresese $\to 0$

Converge

(a) diverges

(b) converges absolutely

(c) converges conditionally

(d) none of the above

$|| \to \sum_{n=1}^{\infty} \frac{1}{\ln(n+1)}$

Compare $\lim_{n\to\infty} \frac{1}{\ln(n+2)} \cdot \ln(n+1)$

$= \lim_{n\to\infty} \frac{\frac{1}{n+1}}{\frac{1}{n+2}}$

$= \boxed{1}$

$\ln(n+1) > \ln(n)$

$\frac{1}{\ln(n+1)} < \frac{1}{\ln(n)}$

root $\frac{n^{\frac{1}{n}}}{\ln n + 10} = \frac{1}{\infty} = 0 < 1$ Con

(15) The series $\sum_{n=1}^{\infty} \frac{n}{(\ln n+10)^n}$

(a) diverges by nth term test
(b) diverges by nth root test
(c) converges by nth root test
(d) converges by nth term test

(16) The sequence $\{a_n = \ln(1 + \frac{1}{n})^n\}$ e^1

(a) converges to 1
(b) converges to e^{-1}
(c) converges to 0 $\ln e^1 = 1$
(d) diverges

(17) The Maclaurin series of $f(x) = x^3 e^x$ is

$e^x = \sum \frac{x^n}{n!}$

(a) $\sum_{n=0}^{\infty} \frac{3^n x^n}{(n+3)!}$

(b) $\sum_{n=0}^{\infty} \frac{3^n x^3}{(n+3)!}$

$= \sum \frac{x^{n+3}}{n!}$

(c) $\boxed{\sum_{n=0}^{\infty} \frac{x^{n+3}}{n!}}$

(d) $\sum_{n=0}^{\infty} \frac{x^{n+3}}{(n+3)!}$

Sandwich

(18) $\lim_{n\to\infty} \frac{8+(-1)^n}{n} =$ $\lim \frac{(-1)^n}{n}$ $\frac{-1}{n}$ $\frac{+1}{n}$ $\to 0$

(a) 0
(b) $\frac{7}{8}$
(c) $\frac{9}{8}$ $\frac{1}{1-1+1} = \frac{1}{2} = \frac{1}{2}/\infty$
(d) DNE

(19) For the series $\sum_{n=1}^{\infty} \frac{5}{n(n+1)}$ $= \frac{5}{2} + \frac{5}{6} + \frac{5}{12}$

(a) $s_n = \frac{(n-4)}{n+1}$, sum = 1 (converges to 1)
(b) $s_n = \frac{n+10}{n}$, sum = 1 (converges to 1)
(c) $s_n = \frac{5n+10}{n+1}$, sum = 5 (converges to 5)
✓✓ (d) $s_n = \frac{5n}{n+1}$, sum = 5 (converges to 5)

$\frac{5}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$

$A = 5$
$B = -5$

$= \frac{5}{n} - \frac{5}{n+1}$

$(5 - \frac{5}{2}) + (\frac{5}{2} - \frac{5}{3})$

$= 5 - \frac{5}{n+1} = \boxed{\frac{5n+5-5}{n+1}}$

(20) One of the following statements is false.

(a) If $\sum a_n$ and $\sum b_n$ converge, then $\sum(a_n + b_n)$ converges.
✓✓ (b) If $\sum a_n$ and $\sum b_n$ converge, then $\sum(a_n\, b_n)$ converges.
(c) If $\sum a_n$ converges and $\sum b_n$ diverges, then $\sum(a_n - b_n)$ diverges.
(d) If $\sum a_n$ diverges, then $\sum(c\, a_n)$ diverges for any nonzero constant c.

(21) The sequence $a_n = \dfrac{2n}{n+1}$ is

$$a_n' = \frac{2(n+1) - 2n}{(n+1)^2}$$

(a) Bounded, convergent, and decreasing.

(b) Unbounded, divergent, and decreasing.

(c) Bounded, convergent, and increasing.

(d) Unbounded, divergent, and decreasing.

$$\frac{2n + 2 - \cancel{2n}}{(n+1)^2} = \frac{2}{(n+1)^2} \text{ Pos.}$$

increas

(22) The general term of the sequence: $\frac{1}{2}, -\frac{1}{6}, \frac{1}{12}, -\frac{1}{20}, \cdots$ is

(a) $\left\{\frac{(-1)^{n+1}}{2n}\right\}_{n=1}^{\infty}$ $\cancel{(-1)^n}$

(b) $\left\{\frac{(-1)^n}{2n}\right\}_{n=1}^{\infty}$

(c) $\left\{\frac{(-1)^n}{n^2+n}\right\}_{n=1}^{\infty}$

(d) $\left\{\frac{(-1)^{n+1}}{n^2+n}\right\}_{n=1}^{\infty}$

(23) The sequence a_n, where $a_1 = 3$ and $a_{n+1} = \dfrac{a_n}{3}$

(a) Converges to 3

(b) Converges to $\frac{1}{3}$

(c) Converges to 0

(d) Diverges.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{1}{3} = \frac{1}{3} < 1$$

(24) The number $5.\bar{4} =$ $= 3 + 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$ Conv $\to 0$

(a) $\frac{4}{99}$ $\text{sum}\left(5 + \frac{4}{10} + \frac{4}{100}\right.$

(b) $\frac{49}{99}$

(c) $\frac{14}{9}$

(d) $\frac{49}{9}$

(25) The series $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{n^2+1}$

$$\frac{\sqrt{n+1}}{n^2+1} \Rightarrow \frac{\sqrt{n}\,\sqrt{1+\frac{1}{n}}}{n^2\left(1+\frac{1}{n^2}\right)}$$

(a) Diverges by direct comparison with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$

(b) Converges by limit comparison with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$

$$= \frac{\sqrt{1+\frac{1}{n}}}{\left(1+\frac{1}{n^2}\right)} \cdot \frac{1}{n^{\frac{3}{2}}}$$

(c) Diverges by direct comparison with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}}$

(d) Converges by limit comparison with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}}$

$$= \frac{1}{n^{\frac{3}{2}}}$$

$$= \lim_{n\to\infty} \frac{\sqrt{1+\frac{1}{n}}}{1+\frac{1}{n^2}}$$

$$= \boxed{1}$$

(26) The series $\sum_{n=1}^{\infty}\left(\frac{\ln n}{n}\right)^n$

$\frac{\ln n}{n} < 1$ $\to$ $\left(\frac{\ln n}{1}\right)$ $= 0$

$1 - \frac{\ln}{n}$

$\lim_{n\to\infty} \frac{\ln n}{n} = 0 < 1$ Converge

(a) Diverges using the nth term test.

(b) Converges using the root test.

(c) Diverges using the ratio test.

(d) ~~None of the above.~~

Conv by

(27) If we estimate the sum of the series $\sum_{n=1}^{\infty}\frac{(-1)^n}{n!}$ using the partial sum S_3, then

(a) $E<0$ and $|E|<\frac{1}{24}$

(b) $E\geq 0$ and $|E|<\frac{1}{24}$

(c) $E<0$ and $|E|<\frac{1}{120}$

(d) $E>0$ and $|E|<\frac{1}{120}$

$P_3 = \frac{-1}{1} + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120}$

$E < 0$

$|E| < \frac{1}{120}$

(28) For the series $\sum_{n=0}^{\infty} n!(x-2)^n$, the radius of convergence is

(a) 0

(b) 1

(c) 2

(d) ∞

$\lim_{n\to\infty} \frac{(n+1)!\,(x-2)^{n+1}}{n!\,(x-2)^n} = \lim (n+1)\,(x-2)$

$= \infty\,(x-2) < 1$

$\boxed{x=2}$

$R=0$

(29) The series $\sum_{n=1}^{\infty}\frac{4^n x^{2n}}{n}$ converges conditionally when

(a) $x=\frac{1}{2}$

(b) $x=-\frac{1}{2}$

(c) $x=\pm\frac{1}{2}$

(d) None of the above.

$\frac{4^n}{n}\,|x^{2n}| < 1$

$\lim_{n\to\infty}\left|\frac{4\,x^2}{n^{\frac{1}{n}}}\right|$

$\frac{4^n\left(\frac{1}{2}\right)^{2n}}{n} \to \frac{4^n\left(\frac{1}{4}\right)^n}{n}$ $x^2 | < 1$ $\sum(-1)^n \frac{1}{n}$

$-\frac{1}{4} < x^2 < \frac{1}{4}$

$\sqrt{-\frac{1}{4}} < x < \boxed{\frac{1}{2}}$

(30) The sum of the series $\sum_{n=1}^{\infty}\frac{(-2)^{n-1}}{4^n}$ is

(a) $\frac{1}{6}$

(b) $\frac{2}{3}$

(c) $-\frac{1}{3}$

(d) $-\frac{4}{3}$

$\frac{(-2)^n}{-2\;\;4^n} = \frac{-1}{2} \times \sum \left(-\frac{1}{2}\right)^n$

$= -\frac{1}{2}\left(\frac{-1/2}{1+\frac{1}{2}}\right)$

$= +\frac{1}{2}\left(\frac{+1/2}{3/2}\right) = \boxed{\frac{1}{2}\left(\frac{1}{3}\right)}$

(31) Given the series $\sum_{n=1}^{\infty}\frac{n^n}{n!}$, then

(a) The series diverges by the ratio test.

(b) The series diverges by the nth term test.

(c) $\lim_{n\to\infty}\frac{n^n}{n!}\neq 0$

(d) All of the above.

$\lim_{n\to\infty} \frac{(n+1)^{n+1}}{(n+1)!}\;\frac{n!}{n^n}$

$= \frac{(n+1)^{n+1}}{n+1}\;\frac{1}{n^n} = \frac{(n+1)^n}{n^n}$

$= \left(\frac{n+1}{n}\right)^n = \left(1+\frac{1}{n}\right)^n$

$= \boxed{e^1} > 1$ div

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32) The sum of the series $\sum_{n=0}^{\infty} \frac{2^{3n}}{3^{2n}}$ is

$= \sum \left(\frac{8}{9}\right)^n$

$= \frac{1}{1-\frac{8}{9}} = \frac{1}{\frac{1}{9}} = 9$

(a) 9

(b) $\frac{1}{9}$

(c) $-\frac{1}{9}$

(d) The series diverges.

33) One of the following series diverges.

(a) $\sum_{n=1}^{\infty} \frac{1}{n^e}$ Conv

(b) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{\ln 2}}$

$\frac{1}{n^{\ln 2}}$ positive, $\to 0$, decrease

(c) $\boxed{\sum_{n=1}^{\infty} \frac{1}{n^{\ln 2}}}$ conv

div ~~Sex~~

$\ln 2 > 1$

(d) ~~$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^e}$~~

$\frac{1}{n^e}$ +, $\to 0$, decrease

$\ln 2 = x$

$2 = e^x$

$2 = (2.17)^x$

$x < 1$

$\ln 2 < 1$

Question **2** (10%). Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-2)^{n+1}}{n4^n}$.

9

1. Find the interval of convergence

Root : $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim \left(\frac{(-1)(-1)^{\frac{1}{n}}(x-2)(x-2)^{\frac{1}{n}}}{n^{\frac{1}{n}}(4)} \right)$

$= \left| \frac{(-1)(x-2)(x-2)^0}{4} \right| = \left| \frac{2-x}{4} \right| < 1$

$-1 < \frac{2-x}{4} < 1 \Rightarrow -4 < 2-x < 4$

$\Rightarrow -6 < -x < 2$

$\boxed{6 > x > -2}$

at $x=6$: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(4)^{n+1}}{n(4)^n}$

$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(4)}{n}$ $\boxed{u_n = \frac{4}{n}}$ Positive decrease $\to 0$ A.S.T $\boxed{\text{Conv.}}$ at $x=6$

at $x=-2$: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-4)^{n+1}}{n(4)^n}$

$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-1)^{n+1}4^{(n+1)}}{n(4)^n} = \sum_{n=1}^{\infty} (-1)^{2n+2}\frac{4}{n}$

A.S.T. Like this $u_n = \frac{4}{n}$ $\boxed{\therefore \text{Conv. at } x=-2}$

The interval $= 6 \geq x \geq -2$

$[-2, 6]$

2. Find the radius of convergence

The Center $= 2$

$-2 \quad 2 \quad 6$ (4, 4)

$\boxed{R = 4}$

3. For what values of x does the series converge absolutely

at $x=6$: $\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{(4)^n(4)}{n(4^n)} = \sum_{n=1}^{\infty} \frac{4}{n}$ div. by P-integral

at $x=-2$: $\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{4}{n}$ div $\boxed{P=1}$ by P-integral $\Rightarrow$ $\boxed{6 > x > -2 \quad]-2, 6[}$

4. For what values of x does the series converge conditionally.

at $\boxed{x = -2, 6}$